SIMILARITY PARAMETERS AT HIGH HYPERSONIC SPEEDS

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Hayes and Probstein [1-2] demonstrated that the flow around an arbitrary body for $M_\infty \to \infty$ reaches a limiting distribution, which, for a given gas with due account of various physico-chemical processes, depends only on the free-stream density ρ_∞ and velocity U_∞ and does not depend on Mach number, static temperature, pressure, etc. This law, known as the Mach number independence principle,* makes it possible to simplify the similarity laws for the flow of a perfect gas at moderate supersonic speeds. Let us postulate that the ratio of specific heats, κ, and the Prandtl number, σ, are constant and that the viscosity, μ, depends on the temperature as follows

$$\mu = cT^n \qquad (c, n = \mathrm{const})$$

Utilizing dimensional analysis and the M independence principle, it can be shown that the dimensionless quantities

$$\varkappa, \quad \sigma, \quad n, \quad T_w/T_0, \quad R_0$$

are the similarity parameters for the flow around geometrically similar, arbitrary bodies with a given distribution of surface temperature, as $M_\infty \to \infty$. Here T_w represents a characteristic surface temperature and T_0

* Literal translation: hypersonic stabilization principle.

the free-stream stagnation temperature. The parameter R_0* is given by

$$R_0 = \frac{\rho_\infty U_\infty l}{\mu_0} \qquad (\mu_0 = cT_0{}^n)$$

where l is a characteristic length of the body. This "effective" Reynolds number depends only on the free-stream density and velocity as contrasted with the ordinary Reynolds number R_∞, which also depends on the static temperature T_∞. For hypersonic speeds R_0 and R_∞ are connected by the relation

$$\frac{R_0}{R_\infty} = \left[\frac{2}{(\varkappa - 1)M_\infty{}^2}\right]^n, \qquad R_\infty = \frac{\rho_\infty U_\infty l}{cT_\infty{}^n}$$

so that an increase in M_∞ for fixed R_∞ leads to a decrease in the effective Reynolds number, i.e. to an increase in viscosity effects.

Hence, instead of the usual similarity for moderate Mach numbers, which requires matching of two parameters M_∞ and R_∞ (or M_∞ and R_0), at large hypersonic speeds it is sufficient to match only the effective Reynolds numbers R_0 (simultaneously with κ, σ, n and T_w/T_0). Because the parameter R_0 was introduced with mildest restrictions on the character of the flow, it must characterize a wide range of phenomena at large hypersonic speeds: occurrence of transition, various interaction effects, separation, etc. In particular, Tsien's low-density parameter $M_\infty/\sqrt{(R_\infty)}$ becomes $R_0{}^{-1/2}$ as $M_\infty \to \infty$.

If under the above assumptions on the nature of the gas one considers the problem of interaction between the boundary layer and the inviscid flow around slender bodies as usually posed [2], one can show that the dimensionless parameters

$$\varkappa, \quad \sigma, \quad n, \quad T_w/T_0, \quad M_\infty\tau, \quad \tau^2\sqrt{R_0} \tag{1}$$

are the similarity parameters for flows around affinely related bodies, τ being the fineness thickness ratio. The parameter $\tau^2\sqrt{(R_0)}$, characterizing this interaction, is found on the basis of the M independence

* Translator's Note: Because the mass flux $\rho_\infty U_\infty$ is preserved across a normal shock, Zhilin's parameter is for practical purposes identical with Reynolds number based on conditions behind a normal shock wave, most often denoted by R_{e2} in U.S. R_{e2} has been used for successful correlations of viscous heat conducting flows around blunt bodies even at moderate supersonic speeds for over a decade.

principle and therefore should be valid for arbitrary hypersonic Mach numbers. Combining parameters in (1), one can form the known parameter [2, 3]

$$\chi = \frac{M_\infty^{2+n}}{\sqrt{R_\infty}} = \left(\frac{2}{\varkappa - 1}\right)^{\frac{n}{2}} \frac{M_\infty^2}{\sqrt{R_0}}.$$

For moderate hypersonic speeds ($M_\infty \tau_\Sigma = 0(1)$; where τ_Σ is the thickness ratio of the body when thickened by the boundary layer) the similarity of flows around affinely related bodies requires matching of parameters $M_\infty \tau$ and χ (or $M_\infty \tau$ and $\tau^2 \sqrt{(R_0)}$). Inasmuch as for $M_\infty \to \infty$ the parameters $M_\infty \tau$ and χ also grow beyond bounds, then for large hypersonic speeds the matching of both parameters is not necessary and only one similarity parameter remains – $\tau^2 \sqrt{(R_0)}$. Clearly, the parameter $\tau^2 \sqrt{(R_0)}$ is more convenient than the Hayes-Probstein [2] parameter for $M_\infty \to \infty$, $\tau \sqrt{(R_b)}$ (where the Reynolds number R_b is based on the viscosity and density at the wall of the body), because the former is determined directly from the parameters of the undisturbed flow.

The effect of low density on the flow is characterized by $R_0^{-1/4}$ (order of the ratio between the flow length to the thickness of the boundary layer).

In flows around affinely related slender bodies the following relations are valid

$$\begin{aligned}
C_p &= \frac{p(x)}{\rho_\infty U_\infty^2} = \tau^2 C_p\left(\frac{x}{l}, \varkappa, \sigma, n, \frac{T_w}{T_0}, M_\infty\tau, \tau^2\sqrt{R_0}\right) \\
C_t &= \frac{t(x)}{\rho_\infty U_\infty^2} = \frac{1}{R_0^{3/4}} C_t\left(\frac{x}{l}, \varkappa, \sigma, n, \frac{T_w}{T_0}, M_\infty\tau, \tau^2\sqrt{R_0}\right) \\
C_q &= \frac{q(x)}{\rho_\infty U_\infty^3} = \frac{1}{R_0^{3/4}} C_q\left(\frac{x}{l}, \varkappa, \sigma, n, \frac{T_w}{T_0}, M_\infty\tau, \tau^2\sqrt{R_0}\right) \qquad (2) \\
\delta^* &= \frac{l}{R_0^{1/4}} \delta\left(\frac{x}{l}, \varkappa, \sigma, n, \frac{T_n}{T_0}, M_\infty\tau, \tau^2\sqrt{R_0}\right)
\end{aligned}$$

Here p, t, and q are pressure, shear stress, and heat flux to the surface; δ^* is the thickness of the boundary layer. For the flow around a flat plate, we have

$$C_p = \frac{1}{\sqrt{R_0}} C_p\left(\frac{x}{l}, \varkappa, \sigma, n, \frac{T_w}{T_0}, \frac{M_\infty^2}{\sqrt{R_0}}\right) \qquad \text{etc.} \qquad (3)$$

When $M_\infty \tau_\Sigma \gg 1$, the M independence principle becomes valid and the parameters containing M_∞ disappear from the similarity relations (2)-(3).

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